

Neutrino mass constraints on β decay

Takeyasu M. Ito¹ and Gary Prézeau²

¹*The Department of Physics and Astronomy, The University of Tennessee, Knoxville, TN 37996
and Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN, 37831*

²*Jet Propulsion Laboratory/California Institute of Technology,
4800 Oak Grove Dr, Pasadena, CA 91109, USA*

(Dated: February 7, 2008)

Using the general connection between the upper limit on the neutrino mass and the upper limits on certain types of non-Standard Model interaction that can generate loop corrections to the neutrino mass, we derive constraints on some non-Standard Model $d \rightarrow ue^-\bar{\nu}$ interactions. When cast into limits on $n \rightarrow pe^-\bar{\nu}$ coupling constants, our results yield constraints on scalar and tensor weak interactions improved by more than an order of magnitude over the current experimental limits. When combined with the existing limits, our results yield $|C_S/C_V| \lesssim 5 \times 10^{-3}$, $|C'_S/C_V| \lesssim 5 \times 10^{-3}$, $|C_T/C_A| \lesssim 1.2 \times 10^{-2}$ and $|C'_T/C_A| \lesssim 1.2 \times 10^{-2}$.

PACS numbers: 23.40.Bw, 14.60.Pq

Historically, nuclear β decay has played an important role in establishing the $V - A$ structure of the electroweak current of the Standard Model (SM). More recently, precision studies of nuclear and neutron β decay have been used to test the SM and to search for what may lie beyond it. ft -values and various angular correlations, for example, have been measured on various nuclear species for small deviations from what the $V - A$ model of weak interactions predicts. These experiments have provided important constraints (for a recent review, see Ref. [1]). With an increased intensity of cold and ultracold neutrons becoming available, increasingly more precise β -decay measurements with free neutrons may probe physics beyond the SM. Neutron β -decay measurements have the special advantage of being free from uncertainties due nuclear structure corrections. The UCNA experiment [2] at the Los Alamos National Laboratory, and the future abBA experiment [3], planned for the Spallation Neutron Source at the Oak Ridge National Laboratory, both aim at precision neutron β -decay measurements that will provide stringent tests of the SM.

On the other hand, various solar, atmospheric and reactor neutrino experiments have provided clear evidence of neutrino oscillation, hence establishing that not all the neutrinos are massless [4, 5, 6]. In addition, the recent remarkable progress in observational cosmology now allows us to study the “Particle Physics” of the early universe through precision measurements of the anisotropy of the Cosmic Microwave Background (CMB). In fact, the most stringent upper limit on the neutrino mass comes from combining *WMAP* [7] and SDSS [8] data.

The fact that the neutrino masses are so much smaller than the other SM fermions – at least six orders of magnitude – together with the fact that the lepton mixing matrix is strikingly different from the quark mixing matrix, may be a window onto new physics. Accordingly, the neutrino mass matrix has become a subject of intensive experimental and theoretical research. At

the same time, the search for new physics through low-energy observables such as muon decay and β decay continues with increasing accuracy. In view of this situation, model-independent connections between the neutrino mass and other low-energy observables would provide valuable guidance in the search for physics beyond the SM.

Recently, an important connection has been pointed out between the neutrino mass and non-SM neutrino-matter interactions in Ref. [9]. That is, if there are non-SM neutrino-matter interactions that involve both right-handed and left-handed neutrinos, they should contribute to the neutrino mass. In Ref. [9], such contributions were evaluated using effective field theory; the requirement that they be smaller than the current neutrino mass limits resulted in non-trivial constraints on various muon decay parameters and the branching ratio of the SM-forbidden $\pi^0 \rightarrow \nu\bar{\nu}$.

In this letter, we extend this treatment to β decay, and obtain order-of-magnitude constraints on some non-SM $n \rightarrow pe^-\bar{\nu}$ interactions. Although we closely follow the notation of Ref. [1], we generalize it to include the possibility of total (and family) lepton-number violation. Therefore, the most general $d \rightarrow ue^-\bar{\nu}$ four-fermion interaction involving both left-handed and right-handed neutrino states can be written as

$$H_\beta = H_{V,A} + H_{S,P} + H_T, \quad (1)$$

$$H_{V,A} = 4 \sum_{\substack{\epsilon, \mu = \{L, R\} \\ l = e, \mu, \tau}} a_{\epsilon\mu} \bar{e} \gamma^\lambda P_\epsilon \nu_l^{(c)} \bar{u} \gamma_\lambda P_\mu d + \text{h.c.}, \quad (2)$$

$$H_{S,P} = 4 \sum_{\substack{\epsilon, \mu = \{L, R\} \\ l = e, \mu, \tau}} A_{\epsilon\mu} \bar{e} P_\epsilon \nu_l^{(c)} \bar{u} P_\mu d + \text{h.c.}, \quad (3)$$

$$H_T = 4 \sum_{\substack{\epsilon = \{L, R\} \\ l = e, \mu, \tau}} \alpha_{\epsilon\mu} \bar{e} \frac{\sigma^{\alpha\beta}}{\sqrt{2}} P_\epsilon \nu_l^{(c)} \bar{u} \frac{\sigma^{\alpha\beta}}{\sqrt{2}} P_\epsilon d + \text{h.c.} \quad (4)$$

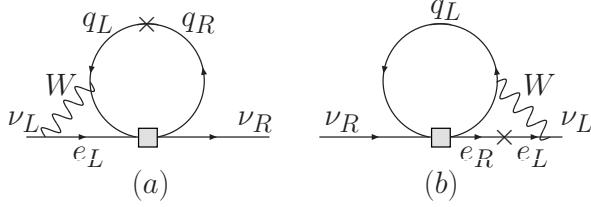


FIG. 1: Two-loop contributions to the neutrino mass generated by chirality-changing non-SM $d \rightarrow ue^- \bar{\nu}$ operators. The \times denote mass insertions. Fig. (a) constrains A_{RR} , A_{RL} , α_{RR} and involves a quark mass insertion $m_q \approx 4$ MeV while Fig. (b) constrains a_{RL} and requires an electron mass insertion, $m_e = 0.511$ MeV.

where, $P_{L,R} = (1 \mp \gamma^5)/2$ is the chirality projection operator, ϵ and μ denote the chiralities of the neutrino and the d quark, respectively, the superscript (c) on the neutrino indicates charge-conjugation for the case where the operators violate total lepton-number conservation, while the subscript l takes into account the possibility of family lepton-number violation. In the SM, $a_{LL} = a_{LL}^{SM} \equiv g^2 V_{ud}/8m_W^2$ and all the other coupling constants $a_{\epsilon\mu}$, $A_{\epsilon\mu}$, and $\alpha_{\epsilon\mu}$ are 0. The $d \rightarrow ue^- \bar{\nu}$ coupling constants $a_{\epsilon\mu}$, $A_{\epsilon\mu}$ and $\alpha_{\epsilon\mu}$ can be related to more conventionally used $n \rightarrow pe^- \bar{\nu}$ coupling constants C_i and C'_i ($i = \{V, A, S, T\}$) [10], as shown in the Appendix. In the SM, $C'_V = -C_V$, and $C'_A = -C_A$.

As discussed in Ref. [9], certain types of non-SM interactions generate contributions to neutrino mass through loop effects. In this case, the A_{RR} -, A_{RL} -, and α_{RR} -type interactions contribute to the neutrino mass through the diagram shown in Fig. 1a, while the a_{RL} -type interaction contributes through the diagram in Fig. 1b [23].

Following Ref. [9], we evaluate the leading log contributions to the neutrino mass from the diagrams in Fig. 1 using dimensional regularization. The result is

$$\delta m_\nu \approx g^2 N_c G_F \bar{a} \frac{m_f M_W^2}{(4\pi)^4} \left(\ln \frac{\mu^2}{M_W^2} \right)^2, \quad (5)$$

where N_c is the number of color degrees of freedom, G_F is the Fermi constant $G_F = 1.166 \times 10^{-5}$ GeV $^{-2}$, m_f is the mass of the fermion for the mass insertion, $g \cong 0.64$ is the SU(2) gauge coupling constant, and μ is the renormalization scale discussed below. $\bar{a} = \{\bar{A}_{RR}, \bar{A}_{RL}, \bar{\alpha}_{RR}, \bar{a}_{RL}\}$ is the “normalized” $d \rightarrow ue^- \bar{\nu}$ coupling, with $\bar{A}_{RR} = A_{RR}/a_{LL}^{SM}$, etc. \bar{a} in principle can have four indices denoting the flavors of incoming and outgoing leptons and quarks. We suppress the indices here, however, for simplicity. We ignored the factors of half or two in the loop integrals associated with different Dirac structures, since we are interested in the orders of magnitude.

The value of μ should exceed the mass of the heaviest particle included in the effective field theory – in our case M_W – while at the same time take into account the scale

at which the onset of new physics might be expected. We choose the renormalization scale to be around 1 TeV, a scale often associated with physics beyond the SM in many particle physics models. Since μ appears in a logarithm, our conclusions do not depend strongly on its precise value. Note that for values of μ below ~ 100 GeV, the decoupling theorem kicks in and large logarithms become suppressed by inverse powers of the weak boson mass. This can be easily seen in the physical renormalization scheme where loop corrections are required to vanish at the renormalization point. In that case, the logarithms will have the generic form $\ln |(\mu^2 - M_W^2)/(Q^2 - M_W^2)|$. At $Q^2 = \mu^2$, this vanishes as required. For small values of Q^2 , large logarithms of the form $\ln(\mu^2/M_W^2)$ appear only for values of $\mu^2 \gg M_W^2$. For $\mu^2 \ll M_W^2$, the large logarithms are suppressed by inverse powers of M_W^2 . It follows that arbitrarily small values of μ cannot be inserted in Eq. (5) which was evaluated for $\mu > M_W$. A more detailed discussion of the dependence of δm_ν on the renormalization scale is given in Ref. [9].

The interactions of the Hamiltonian in Eq. (1) are not gauge-invariant under $SU(2)_L \times U_Y$, although they can arise from gauge-invariant models, e.g., the left-right symmetric and lepto-quark models [1]. Thus, the evaluation of the diagrams of Fig. 1 above the weak scale is not strictly orthodox; in principle, one should evaluate contributions to the neutrino mass within each gauge-invariant model to constrain the relevant parameters appearing in the particular model. The advantage of the current approach stems from the emphasis on the physics that all extensions of the SM that generate the chirality-flipping interactions of Eq. (1) share: they all contain operators that contribute to the neutrino mass resulting in constraints on a class of model parameters. The rough estimates given in Eq. (5) make this novel point in an essentially model-independent way; these estimates are expected to be representative of the constraints imposed on the parameters due to the smallness of the neutrino mass, although this should be checked in specific models. Indeed, the neutrino mass is not required to vanish by gauge-invariance. Therefore, there is no reason to expect cancellations between the diagrams that contribute to m_ν in gauge-invariant models.

We use Eq. (5) to constrain \bar{a} by requiring $\delta m_\nu < m_\nu$ where m_ν is the physical neutrino mass. As in Ref. [9] we adopt the upper limit of 0.71 eV on the sum of the neutrino masses from Ref. [7]. This implies the limit $m_\nu < 0.23$ eV for individual neutrino masses when neutrino oscillation constraints are included [21].

For Fig. 1 (a), from Eq. (5) with $m_f = m_{u,d} \approx 4$ MeV, we obtain $\bar{A}_{RR} < 10^{-3}$, $\bar{A}_{RL} < 10^{-3}$, and $\bar{\alpha}_{RR} < 10^{-3}$. For Fig. 1 (b), with $m_f = m_e$, we obtain $\bar{a}_{RL} < 10^{-2}$. If one takes a neutrino mass limit < 0.04 eV [11] possibly reached by the future Planck mission [12], one obtains $\bar{A}_{RR} < 10^{-4}$, $\bar{A}_{RL} < 10^{-4}$, $\bar{\alpha}_{RR} < 10^{-4}$, and $\bar{a}_{RL} < 10^{-3}$. The obtained constraints on \bar{a} are sum-

marized in Table I together with experimental limits, and constraints on quantities derived from \bar{a} , which we discuss below.

Our limit $\bar{a}_{\text{RL}} < 10^{-2}$ is comparable to the present experimental limit from β decay $\bar{a}_{\text{RL}} < 3.7 \times 10^{-2}$ [13] (see also Ref. [1]).

As seen from the equations in the Appendix, A_{RR} , A_{RL} , and α_{RL} are related to the $n \rightarrow pe^-\bar{\nu}$ coupling constants as follows:

$$2g_S(A_{\text{RR}} + A_{\text{RL}}) = C_S + C'_S, \quad (6)$$

and

$$4g_T\alpha_{\text{RR}} = C_T + C'_T. \quad (7)$$

Our results yield order-of-magnitude constraints of

$$|\tilde{C}_S + \tilde{C}'_S| \lesssim 10^{-3}, \quad (8)$$

and

$$|\tilde{C}_T + \tilde{C}'_T| \lesssim 10^{-2}, \quad (9)$$

where $\tilde{C}_S = C_S/C_V$, $\tilde{C}'_S = C'_S/C_V$, $\tilde{C}_T = C_T/C_A$, and $\tilde{C}'_T = C'_T/C_A$. We used $C_V \cong g_V a_{\text{LL}}^{\text{SM}}$, $C_A \cong g_A a_{\text{LL}}^{\text{SM}}$, and $0.25 \lesssim g_S \lesssim 1$ and $0.6 \lesssim g_T \lesssim 2.3$ [1, 14].

The current experimental limits on C_S and C'_S come from the $e^+\bar{\nu}$ correlation in ${}^{32}\text{Ar}$ β decay [15] and the ft values of super-allowed β decays [16] (An updated analysis gives a similar limit [17]). A combined analysis of data from Refs. [15] and [16] gives a one-standard deviation bound of $|\tilde{C}_S|^2 \leq 3.6 \times 10^{-3}$ and $|\tilde{C}'_S|^2 \leq 3.6 \times 10^{-3}$ [15], which implies $|\tilde{C}_S + \tilde{C}'_S| \lesssim 10^{-1}$. Our constraints are more stringent by two orders of magnitude and are compared with the existing limits in Fig. 2 where it is seen that they are complimentary to the existing limits. Combining our results with the existing limits yields $|\tilde{C}_S| \lesssim 5 \times 10^{-3}$ and $|\tilde{C}'_S| \lesssim 5 \times 10^{-3}$.

For the tensor interaction, the present experimental limit is provided by ${}^6\text{He}$ β decay [18] and the positron polarization of ${}^{14}\text{O}$ and ${}^{10}\text{C}$ β decay [19]. Ref. [18] quotes $(|C_T|^2 + |C'_T|^2)/(|C_A|^2 + |C'_A|^2) < 0.8\%$ (68% C.L.), which implies $|\tilde{C}_T + \tilde{C}'_T| \lesssim 1.6 \times 10^{-1}$. Our results provide a constraint improved by an order of magnitude. Our results are shown in Fig. 3 together with the current experimental limits [24]. When combined with the existing limits, our results yield $|\tilde{C}_T| \lesssim 1.2 \times 10^{-2}$ and $|\tilde{C}'_T| \lesssim 1.2 \times 10^{-2}$.

The initial goal of the UCNA experiment is to measure the β asymmetry parameter A using ultracold neutrons at the 0.2% level. With an implementation of additional detectors, the UCNA experiment also aims to measure the $e^- - \bar{\nu}_e$ angular coefficient a , the $\bar{\nu}_e$ asymmetry parameter B , and the Fierz interference coefficient b , with the following accuracies: $\delta_a/a \leq 3 \times 10^{-3}$, $\delta_A/A \leq 10^{-3}$, $\delta_B/B \leq 10^{-3}$, and $\delta_b \leq 2 \times 10^{-3}$. The abBA experiment aims to measure the same quantities with similar accuracies using a pulsed cold neutron beam.

TABLE I: Constraints on $d \rightarrow ue^-\bar{\nu}$ coupling constants $\bar{a} = \{\bar{A}_{\text{RR}}, \bar{A}_{\text{RL}}, \bar{\alpha}_{\text{RR}}, \bar{a}_{\text{RL}}\}$ obtained from this study (top) and constraints on quantities derived from \bar{a} (bottom) together with current experimental limits.

\bar{a}	Current limits	Limits from this study
$ \bar{A}_{\text{RR}} + \bar{A}_{\text{RL}} $	~ 0.1	$\sim 2 \times 10^{-3}$
$ \bar{\alpha}_{\text{RR}} $	8×10^{-2} (68% c.l.)	$\sim 10^{-3}$
$ \bar{a}_{\text{RL}} $	3.7×10^{-2} (90% c.l.)	$\sim 10^{-2}$
$n \rightarrow pe^-\bar{\nu}$ Coupling	Current limits	Limits from this study
$ \tilde{C}_S + \tilde{C}'_S $	~ 0.1	$\sim 10^{-3}$
$ \tilde{C}_T + \tilde{C}'_T $	1.6×10^{-1}	$\sim 10^{-2}$

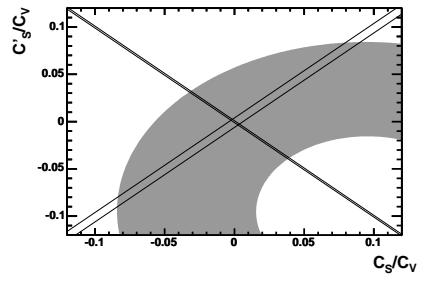


FIG. 2: Constraints on $\tilde{C}_S = C_S/C_V$ and $\tilde{C}'_S = C'_S/C_V$. The narrow diagonal band at -45° is from this work. The annulus gray is a 95% C.L. limit from Ref. [15]. The diagonal band at 45° is a 90% C.L. limit from Ref. [16].

With such precision, it is likely that these free neutron experiments will constrain the scalar interactions by a factor of two better than the current experimental limits [20]. In general β -decay experiments are mostly sensitive to $C_S - C'_S$ and $C_T - C'_T$ through b measurements and $|C_S|^2 + |C'_S|^2$ and $|C_T|^2 + |C'_T|^2$ through a measurements, while our results provide constraints on $C_S + C'_S$ and $C_T + C'_T$, therefore making our analysis and these experiments complimentary.

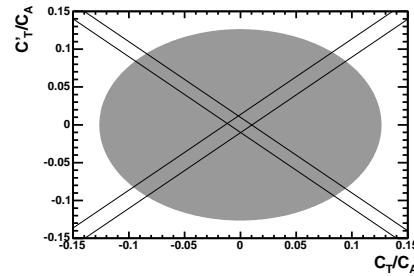


FIG. 3: Constraints on $\tilde{C}_T = C_T/C_A$ and $\tilde{C}'_T = C'_T/C_A$. The diagonal band at -45° is from this work. The gray circle is a 68% C.L. limit from Ref. [18]. The diagonal band at 45° is a 90% C.L. limit from Ref. [19].

The results presented here are quite general and are valid for both Dirac and Majorana neutrinos. As discussed in Ref. [9], there may be cases where the neutrino mass constraints are beaten, for example in the presence of finely tuned cancellations between various Feynman graphs. Also, we do not take into account effects stemming from neutrino mixing with heavy mass eigenstates since in most models they are much heavier than the energy released in β -decay and their emission is kinematically forbidden. If the tritium beta decay limit on neutrino mass ($m_{\nu_e} \lesssim 3$ eV) [21] is used instead of the CMB limit, our analysis still yields limits an order of magnitude more stringent than the current experimental limits for the scalar couplings. We also point out that the KATRIN experiment will have sensitivity down to $m_{\nu_e} \sim 0.2$ eV [22].

In conclusion, we have calculated the leading contributions to the neutrino mass generated by non-SM $d \rightarrow ue^- \bar{\nu}$ interactions that involve right-handed neutrinos. Using the current upper limits on the neutrino mass obtained from CMB measurements, we derived order-of-magnitude constraints on the corresponding coupling constants. When cast into the effective $n \rightarrow pe^- \bar{\nu}$ coupling constants, our results improve over the current experimental constraints on the scalar and tensor coupling constants by more than an order of magnitude. When combined with the existing limits, our results yield $|C_S/C_V| \lesssim 5 \times 10^{-3}$, $|C'_S/C_V| \lesssim 5 \times 10^{-3}$, $|C_T/C_A| \lesssim 1.2 \times 10^{-2}$ and $|C'_T/C_A| \lesssim 1.2 \times 10^{-2}$.

One of the authors (T. M. I) thanks A. Garcia for valuable discussions.

APPENDIX

Here also, we follow the notation of Ref. [1]. Neglecting the induced from factors, the effective $n \rightarrow pe^- \bar{\nu}$ interaction is given by

$$H_\beta^{(N)} \sim H_{V,A}^{(N)} + H_S^{(N)} + H_T^{(N)}, \quad (\text{A.10})$$

where

$$\begin{aligned} H_{V,A}^{(N)} &= \bar{e}\gamma_\lambda(C_V + C'_V\gamma_5)\nu_e\bar{p}\gamma^\lambda n \\ &+ \bar{e}\gamma_\lambda\gamma_5(C_A + C'_A\gamma_5)\nu_e\bar{p}\gamma^\lambda n + \text{h.c.}, \end{aligned} \quad (\text{A.11})$$

$$H_S^{(N)} = \bar{e}(C_S + C'_S\gamma_5)\nu_e\bar{p}n + \text{h.c.}, \quad (\text{A.12})$$

$$H_T^{(N)} = \bar{e}\frac{\sigma^{\lambda\mu}}{\sqrt{2}}(C_T + C'_T\gamma_5)\nu_e\bar{p}\frac{\sigma_{\lambda\mu}}{\sqrt{2}}n + \text{h.c.} \quad (\text{A.13})$$

The $n \rightarrow pe^- \bar{\nu}$ coupling constants C_i and C'_i ($i = \{V, A, S, T\}$) are related to the $d \rightarrow ue^- \bar{\nu}$ coupling constants $a_{\epsilon\mu}$, $A_{\epsilon\mu}$ and $\alpha_{\epsilon\mu}$ ($\epsilon\mu = R, L$) as follows:

$$C_V, C'_V = g_V(\pm a_{LL} \pm a_{LR} + a_{RR} + a_{RL}), \quad (\text{A.14})$$

$$C_A, C'_A = g_A(\pm a_{LL} \mp a_{LR} + a_{RR} - a_{RL}), \quad (\text{A.15})$$

$$C_S, C'_S = g_S(\pm a_{LL} \pm a_{LR} + a_{RR} + a_{RL}), \quad (\text{A.16})$$

$$C_T, C'_T = 2g_T(\pm a_{LL} + a_{RR}), \quad (\text{A.17})$$

where the upper and lower signs are for C_i and C'_i , respectively. The constants $g_i = g_i(0)$ are the $q^2 \rightarrow 0$ values of the nucleon form factors defined by

$$\langle p|\bar{u}\Gamma_i d|n \rangle = g_i(q^2)\bar{p}\Gamma_i n, \quad (\text{A.18})$$

where $i = \{V, A, S, T\}$, and $\Gamma_V = \gamma_\lambda$, $\Gamma_A = \gamma_\lambda\gamma_5$, $\Gamma_S = 1$, and $\Gamma_T = \gamma_5$. CVC predicts $g_V = 1$, $g_A = 1.2695(29)$ [21] (our definition of g_A differs from that adopted by Particle Data Group by the sign).

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 - [23] The a_{RR} -type interaction also contributes to the neutrino mass. We do not consider it here, however, since a stringent constraint on a_{RR} cannot be arrived at by the method discussed in this paper.
 - [24] If one allowed $b \neq 0$ in analyzing the ${}^6\text{He}$ β decay data, one would obtain an annulus for the allowed region as in the ${}^{32}\text{Ar}$ case. However, this would not affect the main conclusion of this paper.